A-Level Maths Summer Work 2023

Please complete and **MARK** this work **before** you return in September. (Answers and hints are included after each set of questions).

Write all you work in a book or on paper in a file **separate from any other work** you have been set, so that you can hand it in for checking **in your first maths lesson.**

Video links:

Algebra: indices Video 17 Indices (numerical) Video 172 Indices: fractional Video 173 Indices: laws of Video 174 Indices: negative Video 175 Algebra: changing the subject Video 7 Algebra: changing the subject advanced Video 8 Algebra: completing the square Video 10 Quadratics: solving (completing the square) Video 267a	Trigonometry introduction: Video 329 Trigonometry missing sides: Video 330 Trigonometry missing angles: Video 331 Trigonometry: sine rule (sides) Video 333 sine rule (angles) Video 334 sine rule (ambiguous case) Video 334a cosine rule (sides) Video 335 cosine rule (angles) Video 336 area of a triangle Video 337

If these videos don't solve your problem, don't hesitate to contact me…

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1.3 Indices

1 Express each of these in the form 2^n where n is an integer.

a $2^3 \times 2^4$ **b** $(2^3)^3$

c 4^5 **d** $(2^4 \times 4^2)^3$

2 Express these as fractions in their simplest terms.

a 4^{-2} **b** 2^{-4}

c 5^{-3} **d** $2^{-1} \times 3^{-3}$

3 Evaluate these.

a $9^{\frac{1}{2}}$ **b** $4^{\frac{1}{2}} \times 27^{\frac{1}{3}}$

c $\dfrac{64^{\frac{1}{3}}}{16^{\frac{1}{4}}}$ **d** $32^{\frac{1}{5}} \times 17^0$

5 By writing 16 as a power of 2, or otherwise, solve the equation $16^x = 32$.

6 Solve these equations.

a $8^x = 16$ **b** $16^x = 64$

c $9^x \times 3^x = 9$ **d** $\dfrac{8^x}{4^{x+1}} = 32$

7 Express these terms in the form ax^n where a is a real number.

a $\dfrac{4x}{2x^2}$ **b** $\dfrac{1}{2x^3}$

c $3x\sqrt{x}$ **d** $\dfrac{\sqrt[3]{x^2}}{4}$

e $\dfrac{2}{\sqrt{x}}$ **f** $\dfrac{3x}{\sqrt[3]{x}}$

g $\dfrac{3\sqrt{x^3}}{6x^2}$ **h** $\dfrac{10x}{\sqrt[4]{x^3}}$

10 **a** Express $\dfrac{3x^3+2}{x^2}$ in the form $ax + bx^n$, where a, b and n are constants.

b Express $\dfrac{2x^2-3x+1}{2x^2}$ in the form $a + bx^{-1} + cx^{-2}$, where a, b and c are constants.

11 Express these as sums of powers of x.

a $\dfrac{(2x+1)(x-1)}{x}$

b $\dfrac{(3x+2)^2}{x^3}$

c $\dfrac{x^2+3x-6}{\sqrt{x}}$

d $\dfrac{(2+\sqrt{x})^2}{x^2}$

12 A curve C has equation $y = \dfrac{(3x+2)(2x+3)}{x^2}$ where $x > 0$.

a Express y in the form $a + bx^{-1} + cx^{-2}$, where a, b and c are constants.

b Explain why, as x increases, the value of y approaches 6.

c Is there a point on this curve with y-coordinate 6?

1.3 Indices

1 **a** 2^7 **b** 2^9 **c** 2^{10} **d** 2^{24}

2 **a** $\frac{1}{16}$ **b** $\frac{1}{16}$ **c** $\frac{1}{125}$ **d** $\frac{1}{54}$

3 **a** 3 **b** 6 **c** 2 **d** 2

4 **a** 8 **b** 9 **c** 32 **d** 27 **e** $\frac{1}{4}$ **f** $\frac{1}{2}$ **g** $\frac{1}{25}$ **h** $\frac{1}{256}$

5 $\frac{5}{4}$

6 **a** $\frac{4}{3}$ **b** $\frac{3}{2}$ **c** $\frac{2}{3}$ **d** 7

7 **a** $2x^{-1}$ **b** $\frac{1}{2}x^{-3}$ **c** $3x^{\frac{3}{2}}$
d $\frac{1}{4}x^{\frac{2}{3}}$ **e** $2x^{-\frac{1}{2}}$ **f** $3x^{\frac{2}{3}}$
g $\frac{1}{2}x^{-\frac{1}{2}}$ **h** $10x^{\frac{1}{4}}$

10 **a** $3x + 2x^{-2}$ **b** $1 - \frac{3}{2}x^{-1} + \frac{1}{2}x^{-2}$

11 **a** $2x - 1 - x^{-1}$ **b** $9x^{-1} + 12x^{-2} + 4x^{-3}$
c $x^{\frac{3}{2}} + 3x^{\frac{1}{2}} - 6x^{-\frac{1}{2}}$ **d** $4x^{-2} + 4x^{-\frac{3}{2}} + x^{-1}$

12 **a** $6 + 13x^{-1} + 6x^{-2}$

b $13x^{-1} = \dfrac{13}{x}$ and $6x^{-2} = \dfrac{6}{x^2}$ so as x increases, these fractions approach 0.
So $6 + 13x^{-1} + 6x^{-2}$ approaches 6.

c Yes: the point $\left(-\frac{6}{13}, 6\right)$

2.1 Basic algebra

2 Factorise fully these expressions.

a $2x^2y + xy^2$ b $10x^3y^2 - 4x^2y^3$

c $3x^4y^2z + 6x^3yz^2$ d $12x^4y^2 + 6x^2y^2 - 9xy$

3 Expand these expressions. Fully factorise answers where appropriate.

a $(2a^2b)^2$ b $(3ab^2)^3 + (3a^2b)^2$

c $(4a^2b^2)^2 - (2ab^3)^2$

4 Rearrange these equations to make the variable shown in square brackets the subject.

a $P = 3(Q + 4)$ [Q]

b $A = \frac{1}{2}(3B - 1)$ [B]

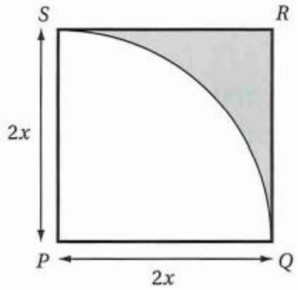
c $R + T = 3(T - 1)$ [T]

d $2(C - D) = 5(1 + 2D)$ [D]

e $U = \frac{1}{3}\sqrt{V + 2}$ [V]

f $M = \frac{\pi}{2}(N - 1)^3$ [N]

6 The diagram shows a square $PQRS$ of side length $2x$ cm. A quarter circle, centre P and radius $2x$ cm, is inscribed inside the square.



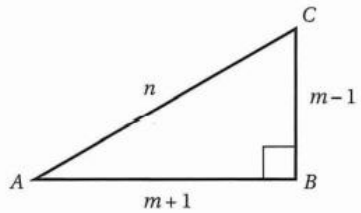
a Show that the area A of the shaded shape is given by the formula $A = 4x^2 - \pi x^2$.

b Make x the subject of this formula.

c Show that the perimeter of the shaded shape is given by the expression

$(4 + \pi)\sqrt{\dfrac{A}{4 - \pi}}$.

7 The diagram shows a right-angled triangle ABC, where $AB = m + 1$, $BC = m - 1$ and $AC = n$.



a If $t = \tan \hat{A}$ show that $m = \dfrac{1 + t}{1 - t}$.

b Find an expression for m in terms of n.

8 Rearrange these formulae to make x the subject.

a $y = (x + 3)^2$

b $y = 4(x - 1)^2 - 1$

c $y = \dfrac{(2x - 5)^2}{3}$

9 $P = \dfrac{2Q + 3}{Q}$

a Show that $P = 2 + \dfrac{3}{Q}$.

b Hence, or otherwise, make Q the subject of the formula $P = \dfrac{2Q + 3}{Q}$.

10 Make the letter indicated in square brackets the subject of these formulae.

a $A = \dfrac{B - 2}{B}$ [B]

b $C = \dfrac{D^2 + 4}{D^2}$ [D]

c $E = \dfrac{5 - 4F^3}{F^3}$ [F]

11 Make the letter indicated in square brackets the subject of these formulae.

a $A = \dfrac{B}{B - 2}$ [B]

b $C = \dfrac{D + 2}{2D + 3}$ [D]

c $E = \dfrac{F^2 + 3}{F^2 + 1}$ [F]

12 Simplify these fractions.

a $\dfrac{x^2 + 3x}{x}$ b $\dfrac{2x^4 + 4x^2}{x^2}$

c $\dfrac{3x^2 - 3x}{x - 1}$ d $\dfrac{x^2 - 2x^3}{2x - 1}$

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2.1 Basic algebra

2 **a** $xy(2x+y)$ **b** $2x^2y^2(5x-2y)$

c $3x^3yz(xy+2z)$ **d** $3xy(4x^3y+2xy-3)$

3 **a** $4a^4b^2$ **b** $9a^3b^2(3b^4+a)$

c $4a^2b^4(2a+b)(2a-b)$

4 **a** $Q=\frac{P}{3}-4$ (or $\frac{P-12}{3}$)

b $B=\frac{2A+1}{3}$ **c** $T=\frac{R+3}{2}$

d $D=\frac{2C-5}{12}$ **e** $V=9U^2-2$

f $N=\sqrt[3]{\frac{2M}{\pi}}+1$

6 **a** Hint: The quarter-circle has area $\frac{1}{4}\pi(2x)^2$

b $x=\sqrt{\frac{A}{4-\pi}}$

c Hint: The quarter-circle has perimeter $\frac{1}{4}(2\pi)(2x)$

7 **a** Hint: Start by expressing t in terms of m using $\tan\hat{A}=\frac{\text{opposite}}{\text{adjacent}}$

b $m=\sqrt{\frac{n^2-2}{2}}$

8 **a** $x=\pm\sqrt{y}-3$

b $x=1\pm\sqrt{\frac{y+1}{4}}$ (or $x=1\pm\frac{1}{2}\sqrt{y+1}$)

c $x=\frac{5\pm\sqrt{3y}}{2}$

9 **a** Hint: Use the rule $\frac{a+b}{c}=\frac{a}{c}+\frac{b}{c}$

b $Q=\frac{3}{P-2}$

10 **a** $B=\frac{2}{1-A}$ **b** $D=\pm\sqrt{\frac{4}{C-1}}$

c $F=\sqrt[3]{\frac{5}{E+4}}$

11 **a** $B=\frac{2A}{A-1}$ **b** $D=\frac{2-3C}{2C-1}$ **c** $F=\pm\sqrt{\frac{3-E}{E-1}}$

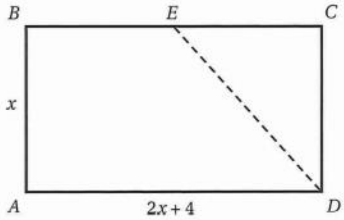
12 **a** $x+3$ **b** $2x^2+4$ **c** $3x$ **d** $-x^2$

2.4 Forming expressions

Unless you are told otherwise, assume all lengths are in centimetres.

1 The diagram shows a rectangle $ABCD$. Point E is the mid-point of BC.

$AB = x$, $AD = 2x + 4$

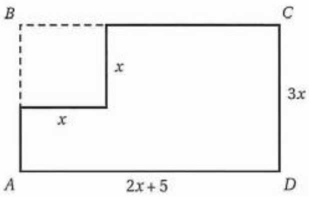


a Find an expression in terms of x for the perimeter of this rectangle.

b Show that the area of the trapezium $ABED$ is given by the formula

Area $= \frac{3}{2}x(x + 2)$.

2 The diagram shows the shape formed when a square of side length x is removed from the rectangle $ABCD$, where $AD = 2x + 5$ and $CD = 3x$.



a Find, in factorised form, an expression for

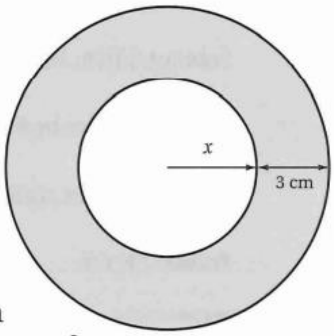
i the perimeter of the shape

ii the area of the shape.

The area of the removed square is 49 cm^2.

b Find the area of the shape.

3 The diagram shows two circles with a common centre. The radius of the smaller circle is x cm. The (shortest) gap between the two circles is 3 cm.

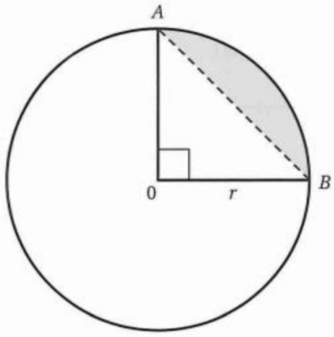


a Find an expression for the circumference of the larger circle. Leave π in your answer.

b Show that the area of the shaded region is given by the formula

Area $= 3\pi(2x + 3)$.

4 The diagram shows a circle with radius r and centre O. Points A and B on the circle are such that triangle AOB is right-angled.



> *Handy hint*
> A sector of a circle looks like a slice of pizza!

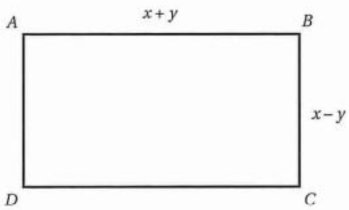
a Show that the perimeter P of the sector OAB is given by the formula

$P = \frac{1}{2}r(4 + \pi)$.

b Find an expression in terms of r for the area between the line AB and the arc AB, as shaded in the diagram. Factorise your answer as far as possible.

5 The diagram shows a rectangle $ABCD$.

$AB = x + y$,
$BC = x - y$,
where $x > y$.

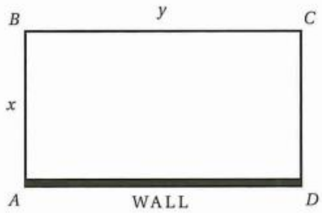


a Find an expression for the perimeter P of this rectangle.

The area of this rectangle is equal to the area of a square with side length y.

b Use this information to show that $x = ky$, stating the exact value of k.

6 An area of land is fenced off using some barbed wire and a wall. In the diagram the wire is represented by the edges AB, BC and CD. The side AD represents the wall, where $ABCD$ is a rectangle.



The total length of barbed wire used is 24 metres.

a Express this information as an equation involving x and y.

b Hence show that the area of this enclosure is given by the formula: Area $= 2x(12 - x)$.

c Find the enclosed area in the case when $ABCD$ is a square.

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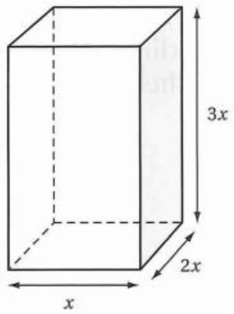
7 The diagram shows a cuboid with dimensions x, $2x$ and $3x$.



a Find an expression for the volume V of this cuboid. Simplify your answer as far as possible.

b Show that the surface area S of the cuboid is given by the formula $S = 22x^2$.

c Express the area of the side shaded in the diagram as a fraction of the surface area. Give your answer in its lowest terms.

The volume of this cuboid is 48 cm³.

d Find the surface area of this cuboid.

8 From a rectangle, four squares of side length x cm are cut from each corner.

Diagram 1 shows the net of the remaining shape.

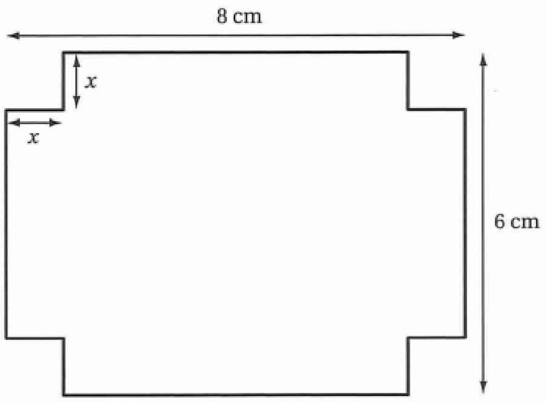


Diagram 1

a Find an expression in terms of x for the area of this net.

The sides of this net are folded at the corners to form a tray (see Diagram 2).

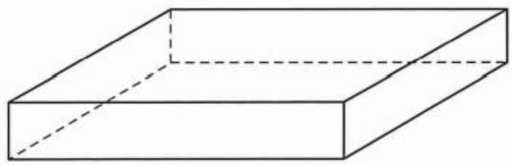


Diagram 2

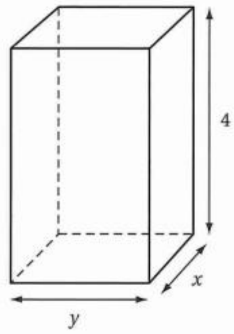
b Show that the volume V of this tray is given by the formula $V = 4x(4 - x)(3 - x)$.

The surface area of this tray is 39 cm².

c Find the value of x.

d Hence calculate the volume of this tray.

9 The diagram shows a cuboid with base dimensions x cm by y cm. The cuboid has height 4 cm.



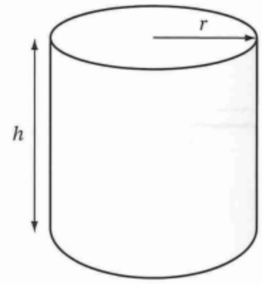
a Find an expression involving x and y for the surface area S of this cuboid.

The volume of this cuboid is 16 cm³.

b Use this information to show that $xy = 4$.

c Find an expression for S in terms of x only.

10 The diagram shows a cylinder with radius r and height h.



a Write down the volume V of this cylinder in terms of r and h.

It is given that the volume of this cylinder is 9π cm³.

b Use this information to express r in terms of h. Simplify your answer as far as possible.

A straight metal rod, which is the longest that can be placed in the cylinder, has length L.

c Show that $L = \sqrt{\frac{36}{h} + h^2}$.

2.4 Forming expressions

1 a $6x + 8$

b Hint: Length $BE = \frac{1}{2}(2x + 4) = x + 2$.

Area of trapezium $= \frac{1}{2}(a + b)h$, where a, b are parallel sides and h is the height.

2 a i $10(x + 1)$ ii $5x(x + 3)$ b $350\ \text{cm}^2$

3 a $2\pi(x + 3)$

b Hint: The area of larger circle is $\pi(x + 3)^2$.

4 a Hint: The quarter-circle has circumference $\frac{1}{2}\pi r$.

b $\frac{1}{4}r^2(\pi - 2)$

5 a $P = 4x$ b $x = \sqrt{2}y$, $k = \sqrt{2}$

6 a $2x + y = 24$

b Hint: Start by rearranging $2x + y = 24$ to make y the subject.

c $64\ \text{m}^2$

7 a $V = 6x^3$

b Hint: The cuboid has 6 faces. The base has area $2x^2$ etc.

c $\frac{3}{11}$ d $88\ \text{cm}^2$

8 a $48 - 4x^2$

b Hint: Label the sides of the tray with its dimensions. e.g. The height of the tray is x cm.

c 1.5 cm d $22.5\ \text{cm}^3$

9 a $S = 2xy + 8x + 8y$

b Hint: Use Volume = base × width × height.

c $S = 8\left(x + \frac{4}{x} + 1\right)$

10 a $V = \pi r^2 h$ b $r = \frac{3}{\sqrt{h}}$

c Use Pythagoras' theorem where L is the hypotenuse.

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3.1 Straight-line graphs

3 **a** Sketch, on the **same** diagram, the line $y - 2x = 1$ and the line $2y - 6x + 1 = 0$.

b Find the distance between the y-intercepts of these graphs.

4 **a** Sketch, on the **same** diagram, the line $y - 3x + 4 = 0$ and the line $3y + x = 6$.

b Find the distance between the x-intercepts of these graphs.

5 Express the equations of these lines in the form $ay + bx + c = 0$, where a, b and c are integers.

a $y = -\frac{1}{2}x - \frac{3}{2}$ **b** $y = \frac{1}{3} - \frac{2x}{3}$

c $y = -\frac{3}{4}x + \frac{1}{2}$ **d** $y = \frac{2}{3}x - \frac{5}{2}$

9 The line L has equation $ay + bx = 10$, where a and b are constants.

The line crosses the y-axis at the point $(0,5)$ and crosses the x-axis at the point $(-2,0)$.

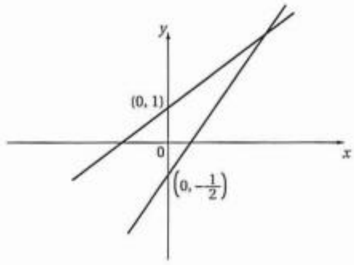
a Using this information, or otherwise, find the value of a and the value of b.

The point $P(4,q)$ lies on this line.

b Find the value of q.

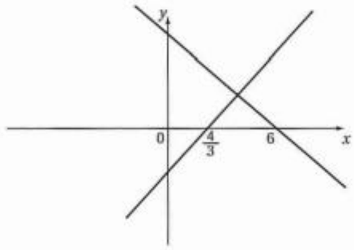
3.1 Straight-line graphs

3 **a**



b $\frac{3}{2}$

4 **a**



b $\frac{14}{3}$

5 **a** $2y + x + 3 = 0$ **b** $3y + 2x - 1 = 0$

c $4y + 3x - 2 = 0$ **d** $6y - 4x + 15 = 0$

9 **a** $a = 2, b = -5$ **b** 15

3.2 The equation of a line

4 Find the equations of the lines passing through these points. Sketch each line on a separate diagram.

a $A(0,-1)$ and $B(5,14)$

b $A(2,5)$ and $B(4,1)$

c $A(-6,-4)$ and $B(10,8)$

d $A\left(\frac{1}{2},2\right)$ and $B(3,12)$

5 **a** A line passes through the points $A(1,5)$ and $B(3,p)$, where p is a constant. Given that the gradient of this line is 4, find the value of p.

b A line passes through the points $A(q,12)$ and $B(6,q)$, where q is a constant. Given that the gradient of this line is -7, find the value of q.

c A line passes through the points $E(r,r+1)$ and $F(8,0)$, where r is a constant.

Given that the gradient of this line is $-\frac{1}{2}$, find the value of r.

6 The line L passes through the points $P(-4,-3)$ and $Q(4,9)$. This line crosses the y-axis at point A and the x-axis at point B.

> *Handy hint*
> Sketch the line L.

a Find an equation for L.

b Write down the coordinates of A.

c Find the coordinates of B.

d Find the area of triangle OAB, where O is the origin.

7 A line passes through the points $S(3,-2)$ and $T(12,-14)$. This line crosses the y-axis at point A and the x-axis at point B.

> *Handy hint*
> Sketch the line.

a Find the coordinates of A and the coordinates of B.

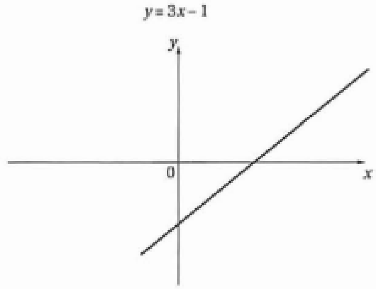
b Show that the distance $AB = \frac{5}{2}$.

8 A line has equation $y = mx - 3$ where m is a constant. The point $A(-5,7)$ lies on this line.

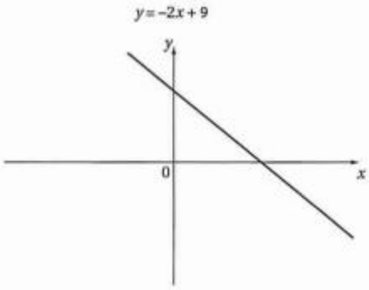
a Find the value of m.

b Determine whether or not the point $B(-7,10)$ lies on this line.

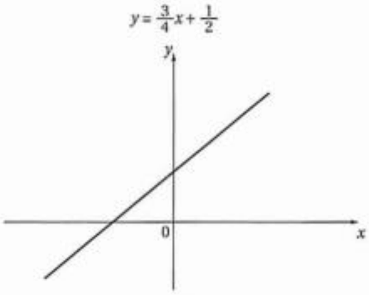
4 **a**



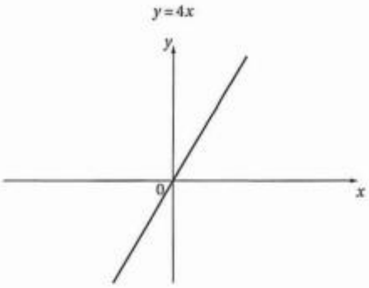
b



c



d



5 **a** 13 **b** 5 **c** 2

6 **a** $y = \frac{3}{2}x + 3$ **b** $(0,3)$ **c** $(-2,0)$ **d** 3

7 **a** $A(0,2)$, $B\left(\frac{3}{2}, 0\right)$

b Hint: Use Pythagoras' theorem on the triangle OAB, where O is the origin.

8 **a** -2

b B does not lie on this line. If you substitute $x = -7$ into the equation, $y = -2(-7) - 3$
$= 14 - 3$
$= 11$

which is not the y-coordinate of B.

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3.4 Parallel and perpendicular lines

1 Write down the gradient of any line which is

a parallel to the line $y = 5 - 3x$

b perpendicular to the line $y = 4x + 1$.

2 Find the gradient of any line which is

a parallel to the line $2y = 5x - 6$

b perpendicular to the line $3y + 4x = 1$.

3 By finding their gradients, show that these pairs of lines are parallel.

a $y = 2x - 4$
$y - 2x + 3 = 0$

b $2y - 3x = 1$
$y = \frac{4 + 3x}{2}$

c $2x + 4y - 3 = 0$
$2y + x + 1 = 0$

5 The line L has equation $y = 2 - 4x$.

a Find the equation of the line which is parallel to L and which passes through the point (0,3).

b Find the equation of the line which is perpendicular to L and which has the same y-intercept as L.

6 The line L has equation $4y - 3x = 11$.

a Find the gradient of L.

b Find the equation of the line which is perpendicular to L and which passes through the point $A(6,-6)$.

Give your answer in the form $ay + bx = c$ for integers a, b and c.

7 The line L has equation $y - 3x + 1 = 0$. The points $A(3,8)$ and $B(-1,k)$, where k is a constant, lie on L.

a Show that $k = -4$.

b Find the equation of the perpendicular bisector of AB. Give your answer in the form $ay + bx = c$, for integers a, b and c.

3.4 Parallel and perpendicular lines

1 a -3 b $-\frac{1}{4}$

2 a $\frac{5}{2}$ b $\frac{3}{4}$

3 a common gradient 2 b common gradient $\frac{3}{2}$
c common gradient $-\frac{1}{2}$

5 a $y = -4x + 3$ b $y = \frac{1}{4}x + 2$

6 a $\frac{3}{4}$ b $3y + 4x = 6$

7 a Hint: Substitute $x = -1$, $y = k$ into the equation.
b $3y + x = 7$

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4.1 Solving a quadratic equation by factorising

4 Use factorising to solve these equations.

a $2x^2 - 9x - 5 = 0$

b $3x^2 + 2 = 5x$

c $2x^2 + 7x = 4$

d $x(x + 4) + 3x + 10 = 0$

e $x(2x - 3) = 2$

f $(x + 3)(x + 5) = 3$

8 a By making the substitution $y = x^2$, express the equation $x^4 - 5x^2 + 4 = 0$ as a quadratic equation in y.

b Find the possible values of y and hence solve the equation $x^4 - 5x^2 + 4 = 0$.

9 Use factorisation to solve these equations. Use the hints if necessary.

a $x - 4\sqrt{x} + 3 = 0$ [Hint: let $y = \sqrt{x}$]

b $x^{\frac{2}{3}} - x^{\frac{1}{3}} - 6 = 0$ [Hint: let $y = x^{\frac{1}{3}}$]

c $2^{2x} - 9 \times 2^x + 8 = 0$ [Hint: let $y = 2^x$]

d $9^x - 10 \times 3^x + 9 = 0$

4.1 Solving a quadratic equation by factorising

4 a $x = 5, x = -\frac{1}{2}$ b $x = 1, x = \frac{2}{3}$

c $x = \frac{1}{2}, x = -4$ d $x = -2, x = -5$

e $x = 2, x = -\frac{1}{2}$ f $x = -2, x = -6$

8 a $y^2 - 5y + 4 = 0$ b $y = 4, y = 1 : x = \pm 2, x = \pm 1$

9 a $x = 1, x = 9$ b $x = -8, x = 27$

c $x = 0, x = 3$ d $x = 0, x = 2$

4.2 Completing the square

1 Express these quadratics in completed square form.

a $x^2 + 6x + 10$ b $x^2 - 4x - 1$

c $x^2 + 8x + 16$

2 a Express $x^2 - 8x + 7$ in the form $(x - p)^2 - q$, where p and q are constants.

b Hence, or otherwise, solve the equation $x^2 - 8x + 7 = 0$.

5 Express these quadratics in completed square form.

a $2x^2 + 12x + 17$ b $3x^2 - 18x + 31$

c $4x^2 - 4x + 3$

4.2 Completing the square

1 a $(x + 3)^2 + 1$ b $(x - 2)^2 - 5$ c $(x + 4)^2$

2 a $(x - 4)^2 - 9$ b $x = 1, x = 7$

5 a $2(x + 3)^2 - 1$ b $3(x - 3)^2 + 4$

c $4\left(x - \frac{1}{2}\right)^2 + 2$

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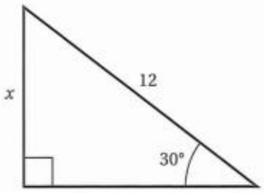
6.1 Trigonometry and triangles

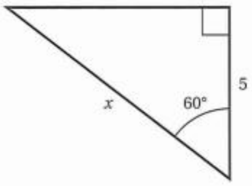
Unless told otherwise, use a calculator and give final answers to 3 significant figures.

1 Use right-angled trigonometry to find the value of x in these diagrams. All lengths are in centimetres.

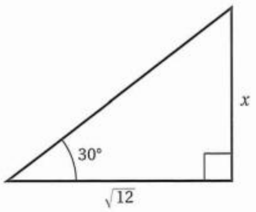
a



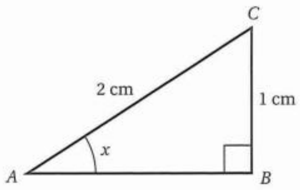
b



c



2 The diagram shows a right-angled triangle ABC. $AC = 2$ cm and $BC = 1$ cm. Angle $CAB = x$.



a Use right-angled trigonometry to show that $x = 30°$.

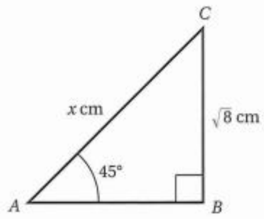
b Use Pythagoras' theorem to show that $AB = \sqrt{3}$.

c Use the triangle to find the **exact** values of

i $\cos 30°$ **ii** $\tan 60°$.

Check each answer on a calculator.

3 a Use right-angled trigonometry to find the value of x in this diagram.



b Write down the length of the side AB.

c Use the triangle to find the value of

i $\tan 45°$

ii $\cos 45°$, giving your answer in simplified surd form.

Check each answer using a calculator.

4 Rearrange these expressions to make the required term the subject.

a $\dfrac{a}{\sin \hat{A}} = \dfrac{b}{\sin \hat{B}}$ for a

b $\dfrac{\sin \hat{A}}{a} = \dfrac{\sin \hat{C}}{c}$ for $\sin \hat{C}$

c $\dfrac{b}{\sin \hat{B}} = \dfrac{c}{\sin \hat{C}}$ for $\sin \hat{B}$

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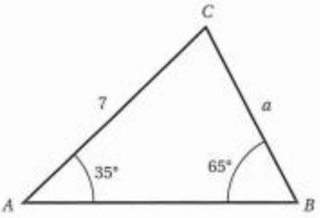
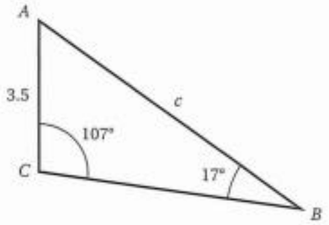
5 In these triangles, all lengths are in centimetres. Use the sine rule to find the length of the side indicated with a lowercase letter.

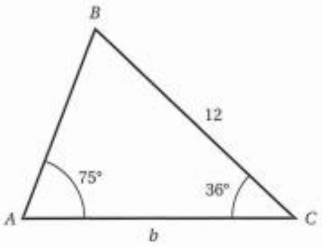
a



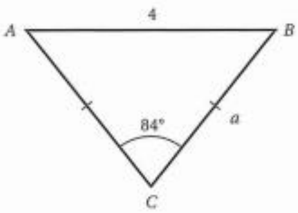
b



c



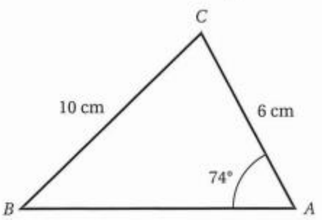
d



Handy hint

The dashes on sides AC and BC mean $AC = BC$.

6 The diagram shows triangle ABC. $AC = 6$ cm, $BC = 10$ cm and angle $BAC = 74°$.

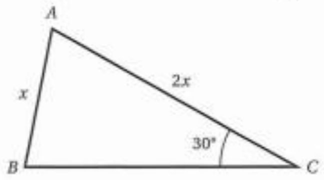


a Show that angle $CBA = 35.2°$ to 3 significant figures.

b Use the sine rule to find the length AB.

7 The diagram shows triangle ABC. $AB = x$, $AC = 2x$ and angle $ACB = 30°$

It is given that $\sin 30° = \frac{1}{2}$.



a Use the sine rule to show that $\sin \hat{B} = 1$.

b Hence show that triangle ABC is right-angled.

c Express the length of the side BC in terms of x, simplifying your answer as far as possible.

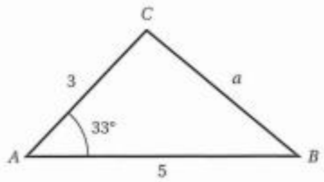
8 Rearrange these cosine rules to make the required term the subject.

a $c^2 = a^2 + b^2 - 2ab\cos\hat{C}$ for $\cos\hat{C}$

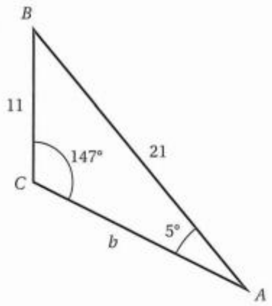
b $b^2 = a^2 + c^2 - 2ac\cos\hat{B}$ for $\hat{B}$

9 In these triangles, all lengths are in centimetres. Use the cosine rule to find the length of the side indicated with a lowercase letter.

a

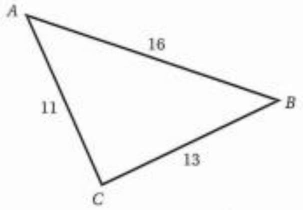


b



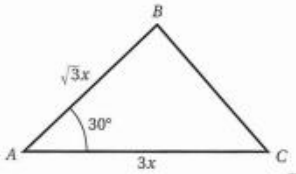
10 In any triangle, the largest angle is opposite the longest side.

a Use the cosine rule to find the largest angle in this triangle.



b Use any appropriate rules to find the other two angles of this triangle.

11 The diagram shows triangle *ABC*.
$AB = \sqrt{3}x$, AC = $3x$ and angle BAC = 30°.



It is given that $\cos 30° = \frac{\sqrt{3}}{2}$.

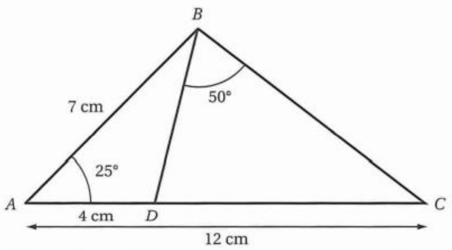
a Show that $BC = \sqrt{3}x$.

Using properties of isosceles triangles, or otherwise,

b find angle *ABC*

c find the area of triangle *ABC*, giving your answer in the form $\frac{3}{4}\sqrt{k}x^2$ where *k* is an integer.

12 The diagram shows the triangle *ABC*, where *AB* = 7 cm, *AC* = 12 cm and angle *BAC* = 25°. Point *D* on *AC* is such that *AD* = 4 cm and angle *DBC* = 50°.

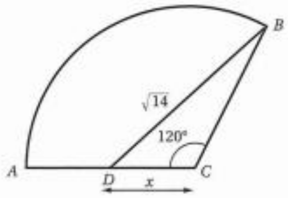


a Show that *BD* = 3.77 cm (to 3 significant figures).

b Find angle *C*.

c Hence, or otherwise, find angle *DBA*.

13 The diagram shows the sector of a circle with centre *C*. Points *A* and *B* lie on this circle. $DB = \sqrt{14}$ cm, where *D* is the mid-point of *AC*. $DC = x$ cm and angle *DCB* = 120°. It is given that $\cos 120° = -\frac{1}{2}$.

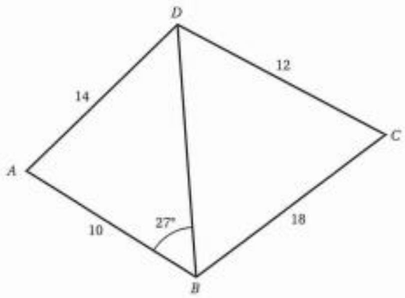


a Show that $x = \sqrt{2}$ cm.

b Find the perimeter of the curved shape *ADB*.

14 The diagram shows a quadrilateral *ABCD*. All lengths are in centimetres. *AB* = 10, *BC* = 18, *CD* = 12 and *DA* = 14.

Angle *ABD* = 27°.



a Show that angle *ADB* = 18.9° (to 3 significant figures).

b Find angle *DCB*.

c Explain briefly why the points *A*, *B*, *C* and *D* cannot all lie on a common circle.

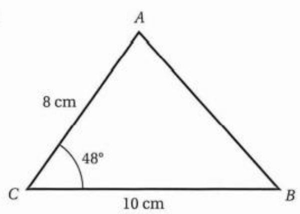
d Show that the diagonal *AC* has length 16 cm to the nearest centimetre.

6.2 The area of any triangle

Unless told otherwise, use a calculator and give final answers to 3 significant figures.

1 Find the area of each of these triangles.

a



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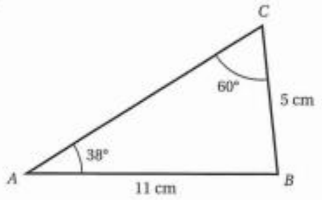
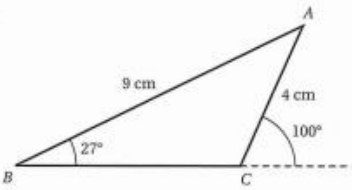
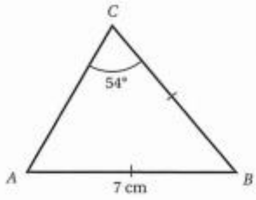
b



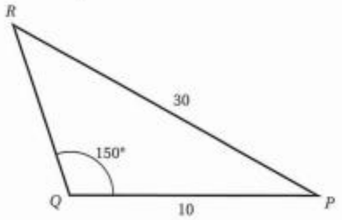
c



2 Find the area of this isosceles triangle.



3 The diagram shows triangle PQR.
$PQ = 10$, $PR = 30$ and angle $PQR = 150°$.
All lengths are in centimetres.



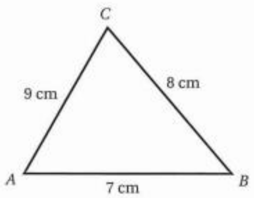
a Use the sine rule to show that $\sin \hat{R} = \frac{1}{6}$.

b Hence find angle QRP.

c Show that triangle PQR has area 52.3 cm^2 to 3 significant figures.

4 Find the area of an equilateral triangle whose perimeter is 15 cm.

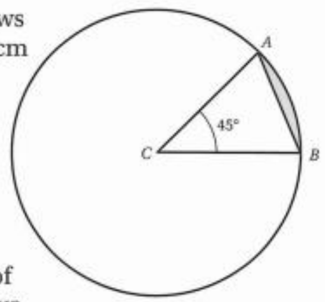
5 In triangle ABC,
$AB = 7$ cm
$BC = 8$ cm
$AC = 9$ cm.



a Use the cosine rule to show that angle $ACB = 48.2°$ (to 3 significant figures).

b Hence find the area of this triangle.

6 The diagram shows a circle, radius 8 cm and centre at point C.
Points A and B on the circle are such that angle $ACB = 45°$.

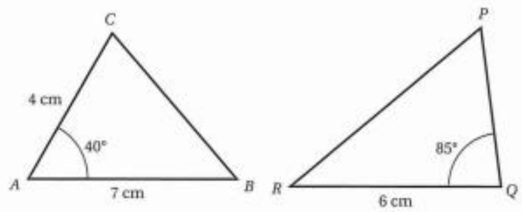


a Find the area of this circle. Leave π in your answer.

b Find the area of triangle ABC.

c Show that the shaded segment between this circle and the line AB has area 2.5 cm^2 (to 1 decimal place).

7 The triangles ABC and PQR shown in the diagram have equal areas.

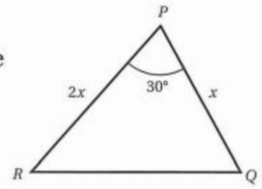


a Find the area of triangle ABC.

b Hence find the length of the side QP.

c Show that the perimeter of triangle PQR is 15.5 cm (to 3 significant figures).

8 The diagram shows the triangle PQR where $PQ = x$, $PR = 2x$ and angle $QPR = 30°$.
All lengths are in centimetres.



It is given that $\sin 30° = \frac{1}{2}$.

a Show that the area A of this triangle is given by the formula $A = \frac{1}{2}x^2$.

It is given that the area of this triangle is 18 cm^2.

b Find the length of the side PQ.

c Hence show that the base RQ of this triangle has length 7.44 cm (to 3 significant figures).

d Find the length of the shortest line from P to the side RQ.

6.1 Trigonometry and triangles

1 **a** 6 cm **b** 10 cm **c** 2 cm

2 **a** Hint: Use $\sin x = \frac{\text{opposite}}{\text{hypotenuse}}$

b Hint: Use $AB^2 = AC^2 - BC^2$.

c **i** $\frac{\sqrt{3}}{2}$ **ii** $\sqrt{3}$

3 **a** 4 **b** $\sqrt{8}$ cm **c** **i** 1 **ii** $\frac{\sqrt{2}}{2}$

4 **a** $a = \frac{b \sin \hat{A}}{\sin \hat{B}}$ **b** $\sin \hat{C} = \frac{c \sin \hat{A}}{a}$ **c** $\sin \hat{B} = \frac{b \sin \hat{C}}{c}$

5 **a** 4.43 cm **b** 11.4 cm **c** 11.6 cm **d** 2.99 cm

6 **a** Hint: Use the sine rule $\frac{\sin \hat{B}}{b} = \frac{\sin \hat{A}}{a}$. **b** 9.82 cm

7 **a** Hint: Rearrange $\frac{\sin \hat{B}}{2x} = \frac{\sin 30}{x}$ to make $\sin \hat{B}$ the subject.

b Hint: Find $\sin^{-1}(1)$. **c** $\sqrt{3}x$

8 **a** $\cos \hat{C} = \frac{a^2 + b^2 - c^2}{2ab}$

b $\hat{B} = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2ac}\right)$

9 **a** 2.97 cm **b** 12.4 cm

10 **a** $\hat{C} = 83.2°$ **b** $\hat{A} = 53.8°$, $\hat{B} = 43.0°$

11 **a** Hint: Use the cosine rule $a^2 = b^2 + c^2 - 2bc \cos \hat{A}$.

b 120°

c $\frac{3\sqrt{3}}{4}x^2$ (Hint: The perpendicular from B bisects the base AC.)

12 **a** Hint: Use the cosine rule on triangle BAD.

b 21.2° **c** 83.8°

13 **a** Hint: Use the cosine rule on triangle BCD, where $BC = 2x$.

b 11.1 cm

14 **a** Hint: Use the sine rule on triangle ADB.

b 93.0°

c Opposite angles in a cyclic quadrilateral sum to 180°.

Since $\hat{A} + \hat{C} = 134.1° + 93.0° = 227.1$, $ABCD$ cannot be a cyclic quadrilateral.

Hence all four points cannot lie on a common circle.

d Hint: Start by finding angle CDB and then use the cosine rule on triangle ADC.

6.2 The area of any triangle

Where appropriate, answers are given to 3 significant figures unless stated otherwise.

1 **a** 29.7 cm² **b** 27.2 cm² **c** 17.2 cm²

2 23.3 cm²

3 **a** Hint: Rearrange the sine rule $\frac{\sin \hat{R}}{10} = \frac{\sin 150°}{30}$ to make $\sin \hat{R}$ the subject.

b 9.59° **c** Hint: Use the formula $\frac{1}{2}qr \sin \hat{P}$.

4 10.8 cm²

5 **a** Hint: Rearrange $c^2 = a^2 + b^2 - 2ab \cos \hat{C}$ to make $\cos \hat{C}$ the subject.

b 26.8 cm²

6 **a** 64π cm² **b** 22.6 cm²

c Hint: The sector CAB has area $\frac{1}{8} \times \pi(8)^2$.

7 **a** 9.00 cm² **b** 3.01 cm

c Hint: Start by using the cosine rule to find the length of the side PR.

8 **a** Hint: Simplify $\frac{1}{2}(x)(2x) \sin 30°$.

b 6 cm

c Hint: Use the cosine rule $p^2 = q^2 + r^2 - 2qr \cos \hat{P}$.

d 4.84 cm

9 **a** 49.8 cm²

b Hint: the sector has area $\frac{17}{72} \times \pi\,(10)^2$.

10 **a** Hint: Start by finding angle BCA and then use the sine rule.

b 47.3 cm² **c** 35.6 cm

11 **a** Hint: Start by finding the length AE by using Pythagoras' theorem on triangle ABE. Then use the cosine rule on triangle AEF.

b $\frac{5}{2}$ cm²

c Hint: Start by calculating angle BEA using right-angled trigonometry. Then find angle CEF.

d $\frac{104}{25}$ cm²