A-level Further Mathematics Transition work

Please make sure you have done your best to understand and work through the material on radian measure. This will be helpful when we start further maths A-level in September.

The questions below are just for fun. If you are doing further maths you must like it!

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- The following questions are all quite hard!
- The questions with an asterisk (*) are the hardest.
- None of the questions **require** knowledge beyond what you learnt for GCSE, but the questions are intended to give you a taste of how it feels to tackle long, difficult questions. You have chosen A-level Further Maths, so, you should enjoy solving long difficult maths problems!
- Many of the questions are structured so that earlier parts help or give a clue to later parts.
- You may spend over an hour on some questions.
- You may get completely stuck on some questions.
- **You should not need to use a calculator for any of these questions.**

If you want a hint on a question, or just want to check an answer email me:
dcrocker@coombedean.co.uk
If you would like me to take a look at what you have done, take a photo of you working and sent it to me.

1

(i) Simplify $\sqrt{50} + \sqrt{18}$.

(ii) Express $(3 + 2\sqrt{5})^3$ in the form $a + b\sqrt{5}$ where a and b are integers.

(iii) Expand and simplify

$$(1 - \sqrt{2} + \sqrt{6})^2 .$$

(iv) **(a)** Expand and simplify $(1 + \sqrt{2})^2$.
(b) Find all real values of x that satisfy

$$x^2 + \frac{4}{x^2} = 12 .$$

Leave your answers in the form of surds.

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2

(i) Solve the equation:

$$\frac{2}{x+3} + \frac{1}{x+1} = 1\,.$$

(ii) Find the value(s) of b for which the following equation has a single (repeated) root.

$$9x^2 + bx + 4 = 0\,.$$

(iii) Find the range of (real) values of c for which the following equation has no real roots:

$$3x^2 + 5cx + c = 0\,.$$

Probably the safest way of dealing with inequalities is to sketch a graph.

3*

In this question a and b are distinct, non-zero real numbers, and c is a real number.

(i) Show that, if a and b are either both positive or both negative, then the equation

$$\frac{x}{x-a} + \frac{x}{x-b} = 1$$

has two distinct real solutions.

(ii) Show that, if $c \neq 1$, the equation

$$\frac{x}{x-a} + \frac{x}{x-b} = 1 + c$$

has exactly one real solution if

$$c^2 = -\frac{4ab}{(a-b)^2}\,.$$

Show that this condition can be written

$$c^2 = 1 - \left(\frac{a+b}{a-b}\right)^2$$

and deduce that it can only hold if $0 < c^2 \leqslant 1$.

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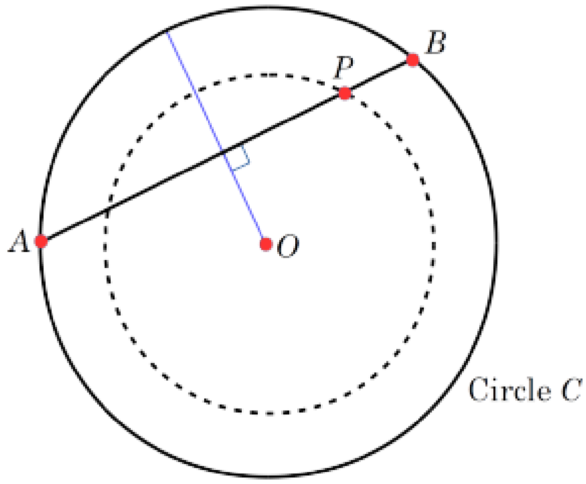
4

The diagram shows a circle C with centre O, and a rod AB the ends of which can slide round the circle C (so that AB is a chord of C). The radius of the circle is R and the length of the rod is $2a$.

As the rod slides round C the point P, which is a fixed distance b from the centre of the rod, traces out a circle with centre O of radius r.



Show that the area between the two circles is $\pi(a^2 - b^2)$.

5*

(i) Simplify $(2x-3)^2-(x-1)^2$, giving your answer in factorised form. Check your answer by evaluating it for $x = 1$ and $x = 2$.

(ii) Simplify

$$\frac{x}{x^2-y^2} - \frac{y}{(x-y)^2} - \frac{1}{x+y}.$$

Hence find the possible values of x and y for which $\dfrac{x}{x^2-y^2} - \dfrac{y}{(x-y)^2} - \dfrac{1}{x+y} = 0.$

(iii) Show that

$$\sqrt{1+x^2} - x = \frac{1}{\sqrt{1+x^2}+x}.$$

Deduce that if x is very large, then $\sqrt{1+x^2} - x$ is approximately equal to $\dfrac{1}{2x}$.

(iv) Simplify $(x^2 - \sqrt{2}x + 1)(x^2 + \sqrt{2}x + 1)$. Hence find the solutions to $x^4 + 1 = 0$.

Your answer will involve $\sqrt{-1}$ (written as i).

Okay, you may have spotted a flaw in bullet point 3 at the beginning. You can leave out part (iv) if $i = \sqrt{-1}$ upsets you, but you may want to google it.

6

(i) Sketch the line $y = x + 1$ for $-2 \leqslant x \leqslant 2$.

What is the greatest value of $x + 1$ in this range?

(ii) Sketch the line $y = -2x + c$ for $-2 \leqslant x \leqslant 2$.

Show that the greatest value of $-2x + c$ in this range is $4 + c$. What is the least value?

(iii) Sketch $y = mx + 1$ for $-2 \leqslant x \leqslant 2$ in the cases $m > 0$, $m = 0$ and $m < 0$.

What are the greatest and least values of $mx + 1$ in each case?

(iv) Sketch the curve (parabola) $y = (x - 1)^2$ for $-2 \leqslant x \leqslant 2$.

What are the greatest and least values of $(x - 1)^2$ in this range?

Be careful here: the minimum value is **not** at one of the end points.

(v) Sketch the curve $y = (x - 3)^2$. What are the greatest and least values of $(x - 3)^2$ for $-2 \leqslant x \leqslant 2$?

(vi) Write the expression $x^2 - 8x + 21$ in the form $(x + a)^2 + b$. Hence sketch the curve $y = x^2 - 8x + 21$ and find the greatest and least values of $x^2 - 8x + 21$ in the range $0 \leqslant x \leqslant 5$.

(vii) Sketch the curve $y = x^2 + 2kx$ for $-2 \leqslant x \leqslant 2$, where $-2 < k < 2$.

What are the greatest and least values of $x^2 + 2kx$ for $-2 \leqslant x \leqslant 2$?

What would your answers be if $k > 2$?

Use the same techniques as in part **(vi)** to help you sketch the curve.

7*

(i) Find the greatest and least values of $bx + a$ for $-10 \leqslant x \leqslant 10$, distinguishing carefully between the cases $b > 0$, $b = 0$ and $b < 0$.

(ii) Find the greatest and least values of $cx^2 + bx + a$, where $c \geqslant 0$, for $-10 \leqslant x \leqslant 10$, distinguishing carefully between the cases that can arise for different values of b and c.

8

Five children (Ahmed, Bachendri, Charlie, Daniel and Emily) raced each other. First they raced to the spreading chestnut tree, and then they raced back to their starting point. The following facts are known:

(i) Ahmed was fourth in the race to the tree.

(ii) The person who was last to the tree managed to win the race back.

(iii) The person who won the race to the tree was third on the way back.

(iv) The person who was third in the race to the tree was second on the way back.

(v) Bachendri was fourth on the way back.

(vi) Charlie reached the tree before Daniel.

(vii) Charlie got back to the start before Emily.

For each race (to the tree and back again), write down the order in which the children finished.

9

(i) *For this question, you need a *good* diagram — nice and big. You may find it helpful to use letters for the sizes of some of the angles; for example, you might want to write 'Let $\angle APO = x$'.*

Let A, B and P be points on the circumference of a circle with centre O, such that O lies inside the triangle ABP. What sort of triangle is APO?

By considering the triangles APO and BPO, prove that

$$\angle AOB = 2\angle APB$$

(where $\angle APB$ is acute and $\angle AOB \leqslant 180°$).

(ii) Prove the same result in the case where O does not lie inside the triangle ABP and $\angle APB$ is acute (and $\angle AOB \leqslant 180°$).

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10 In this question you may find it helpful to use the factor theorem.
If you type "factor theorem" into the search bar on youtube you will find loads of explanations.

(i) Factorise $x^2 - 3x - 4$. Sketch the graph $y = x^2 - 3x - 4$ and hence find the range(s) of values of x for which $x^2 - 3x - 4 > 0$.

(ii) Show that $x = 3$ is a root (solution) of the equation $x^3 - 2x^2 - 5x + 6 = 0$.

Find two more integer roots and write $x^3 - 2x^2 - 5x + 6$ as a product of three linear factors.

Sketch the graph $y = x^3 - 2x^2 - 5x + 6$. Use your sketch to find the ranges of values of x for which $x^3 - 2x^2 - 5x + 6 \leqslant 0$

(iii) Factorise $x^2 - 3x + 2$. Use your answer to help you factorise $x^2 - 3xy + 2y^2$.

Show that if $x^2 - 3xy + 2y^2 = 0$, then the point with coordinates (x, y) lies on one (or both) of two straight lines (you should give the equations of the lines). Sketch the lines on the same set of axes.

(iv) On a sketch, shade the regions of the (x,y) plane in which $x^2 - 3xy + 2y^2 \leqslant 0$.

One technique you can use to determine which regions you want is to pick a point in each region (**not** on the boundary lines) and check to see if $x^2 - 3xy + 2y^2 \leqslant 0$ is true for this point.